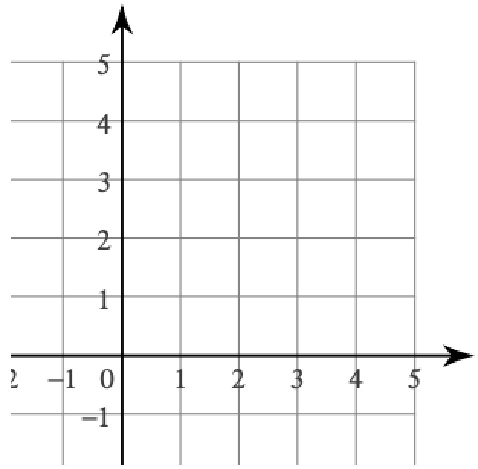
Limits of Functions - Graphically

***Intro Question*:** Consider the function $f(x) = \frac{x^2 - 4}{x - 2}$. What is $f(2)$?

BUT there's more to the story if we look on either side of $x = 2$ ...

x	*y*
1.8	
1.9	
1.99	
1.999	
2	

x	*y*
2.1	
2.01	
2.001	
2.0001	
2	



ONE-SIDED Limits

The limit of *f(x)* as *x* approaches 2 *from the left* is _____. In Notation:

The limit of *f(x)* as *x* approaches 2 *from the right* is _____. In Notation:

Limit of a Function

THE LIMIT of *f(x)* as *x* approaches 2 is _____. In Notation:

Evaluating a Function

The function at *x* = *2* is indeterminant. In Notation:

Super Important: The limit at *x* = *a* doesn't have to match *f(a)* to exist!!

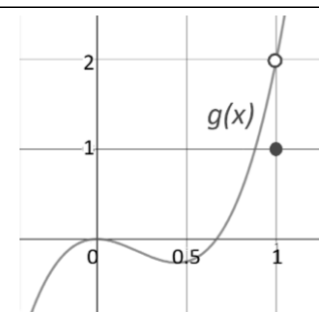
***Example #1*:** When the limit as *x*→ *a* doesn't match *f(a).*

A. $\lim_{x \to 1^-} g(x)$

B. $\lim_{x \to 1^+} g(x)$

C. $\lim_{x \to 1} g(x)$

D. $g(1)$



Important THE Limit of a function at *x* = *a* only exists if the left and right-hand limits *are equal to the same constant.* (Even if $f(a)$ doesn't equal that constant.)

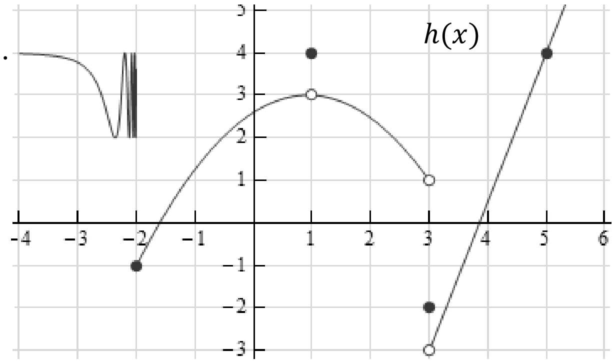
$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L,$$ where *L* is a ***constant***

We say THE limit does not exist (D.N.E) if… ● the left-hand and right-hand limits are not equal

● the outputs oscillate as x approaches a * $f(x)$ approaches positive or negative infinity, but it's still a description

Example#2: The graph represents a SINGLE function, $h(x)$.

How do we know it's a function?



Based on the graph of $h(x)$, find the following values if they exist. If they do not exist, explain why.

A. $\lim_{x \to -2^-} h(x)$	B. $\lim_{x \to -2^+} h(x)$	C. $\lim_{x \to -2} h(x)$	D. $h(-2)$
E. $\lim_{x \to 1^-} h(x)$	F. $\lim_{x \to 1^+} h(x)$	G. $\lim_{x \to 1} h(x)$	H. $h(1)$
I. $\lim_{x \to 3^-} h(x)$	J. $\lim_{x \to 3^+} h(x)$	K. $\lim_{x \to 3} h(x)$	L. $h(3)$
M. $\lim_{x \to 5^-} h(x)$	N. $\lim_{x \to 5^+} h(x)$	O. $\lim_{x \to 5} h(x)$	P. $h(5)$

Infinite Limits

Let f be a function defined on both sides of a, except possibly at a, then $f(x)$ has an ***infinite limit*** as x approaches a if the values of $f(x)$ become arbitrarily large positive or negative from at least one side of a

$$\lim_{x \to a} f(x) = \infty \qquad OR \qquad \lim_{x \to a} f(x) = -\infty$$

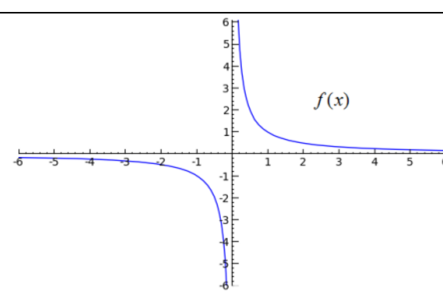
* *NOTE: This does not mean the limit exists in these cases!!*

Note: $x = a$ is a **vertical asymptote** of $f(x)$ if $\lim_{x \to a^-} f(x) = \pm\infty$ OR $\lim_{x \to a^+} f(x) = \pm\infty$ OR $\lim_{x \to a} f(x) = \pm\infty$.

Example#3: Find the following limits.

A. $\lim_{x \to 0^-} f(x)$

B. $\lim_{x \to 0^+} f(x)$



*Can you think of another function that has infinite limits?

(THINK-PAIR-SHARE)

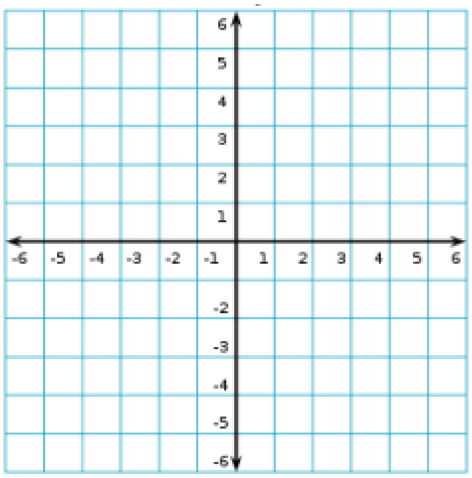
Sketching Functions based on Limit Characteristics

If only given limits and/or a couple of function values, there is NOT one correct answer. But we must be able to translate the information in either direction (Given: *Sketch* → Find: *Limits* *OR* Given: *Limits* → Find: *Sketch*).

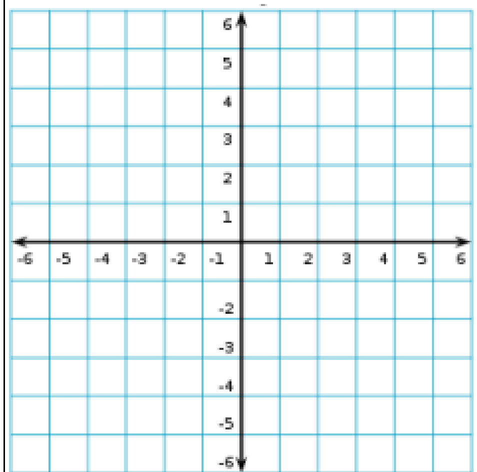
1st: Plot the defined points. **2nd:** Draw points/holes at the limit values. **3rd:** Read carefully to finish sketch conditions

Example 8: Sketch the graph of a function with the following characteristics:

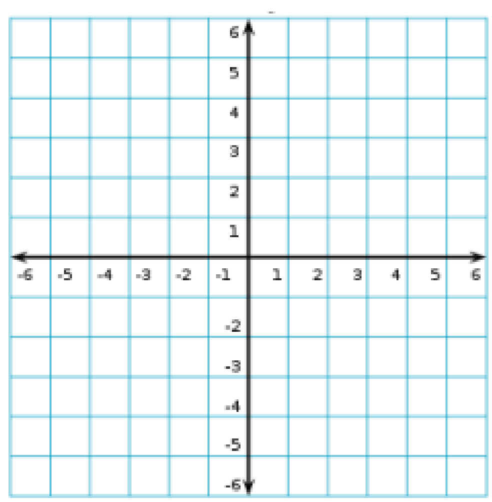
A. $\lim_{x \to -3} f(x) = 2,\ f(-3) = -2$



B. $\lim_{x \to 4^-} f(x) = -1,\quad \lim_{x \to 4^+} f(x) = 3,\ f(4) = 1$



C. $\lim_{x \to -2^-} f(x) = 5,\quad \lim_{x \to -2^+} f(x) = -2,$
$\lim_{x \to 1} f(x) = -\infty,\quad \lim_{x \to 3^+} f(x) = 2,\ f(3) = -1$



Graphing with Limitless Imagination!

Step 1: You will create a sketch of a function on a graph with the following criteria

- At least four features discussed from today
  - Limits from right or left side (preferably at least one where they differ from each side)
  - Two-Sided limits
  - Infinite Limits
  - Additional Limits that do not exist
- At least 2 points on the graph (opened or closed)

Step 2: After your drawing satisfies the 6 criteria, then choose a name for your graph and write out the constraints ***using mathematical notation***.

Step 3: Exchange your rules with a classmate. Make sure to hide your initial drawing to keep the mystery going!

Step 4: Use your partner's limits criteria to sketch a function meeting the provided constraints.

Step 5: Pass it back to your partner and check of the criteria they met.

Step 6: Lastly, please give at least two bits of feedback.

SPECIFIC FEEDBACK

- Put a check mark next to each criteria your partner met.
- Circle the ones that were not met. If you can specify where on the graph they attempted to meet this criteria and connect with an arrow, that would be most useful for your partner's learning expeience (and your own).

GENERAL FEEDBACK

- One positive comment on something you like that your partner did.
- One adjustment or recommendation that could help your partner improve.