

Guide for Using *Thank You Notes: A Children's Story about Exponential Growth*

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Overview

Children's lives have been directly impacted by the exponential-like emergence of cases of COVID-19. They also likely will experience exponential-like growth related to global climate change during their lifetimes. This book provides an emotionally positive example of exponential growth as a way to develop conceptual and mathematical understandings of this phenomenon. Also included is an example of the impact of limited resources.

Intended for use in grades 3-5, this book engages students in developing a mathematical understanding of exponential growth. It uses upper elementary students' skills in multiplying by 2, adding multi-digit numbers, and creating bar graphs. The story integrates mathematics, literacy, social studies, science, and the visual arts as well as addressing many aspects of the US national standards in these curricular areas.

The protagonists are children engaged in an activity that upper elementary children can envision doing themselves, writing thank you notes. The story begins with questions about ways people are helping one another in their families, neighborhoods, and communities. A child, Jasmine, decides to write a thank you note and the next day asks a friend to write one too. Each day, each note writer asks a sibling or a friend to write one. Exponential growth occurs because the number of thank you notes written each day doubles.

The text invites readers to participate in the thinking as the children ponder how many thank you notes will be written later in the week, if each writer continues inviting someone to join them each day. They also ponder how many envelopes they will need by the end of the week ($1+2+4+8+16+32+64 = 127$). A parent agrees to order that many.

The children next contemplate continuing to write thank you notes for a second week, with each writer inviting someone to join them each day. How many envelopes will be needed then? ($128+256+512+1024+2048+4096+8192 = 16,256!$) A parent objects to ordering that many and instead offers to order 4 boxes of 100 envelopes each. How many more days can the children write notes if limited to only 400 more envelopes? This is an engaging way to think about what exponential growth means in terms of how the low initial stages can quickly become overwhelming and exceed limited resources.

This guide provides suggestions for teachers and parents interested in helping children understand what exponential growth is in a context separate from its implications during a pandemic or natural disaster. Also included are connections to US standards in mathematics, English language arts, social studies, science and the visual arts as well as to other contexts illustrating exponential growth. An outline summarizing each page may help teachers and parents plan how to use this book. Also provided is a verbatim text, page by page.

Suggestions for Teachers and Parents

We envision this book as the setting for conversations that evolve according to the children's interests and emerging understandings. These suggestions focus upon ways to ask open-ended questions, engage children in thinking together about many aspects of exponential growth, and encourage them to write and illustrate their own stories. This guide also makes connections to national standards in a variety of curricular areas in grades 3-5 and suggests ways to explore other contexts illustrating exponential growth.

Asking open-ended questions

This book is intended for reading *with* children rather than *to* children. Reading *with* children means engaging children in conversations about what they are thinking as you read. Reading a picture book *with* very young children, for example, means asking open-ended questions such as “what do you see here?” rather than talking *to* children about what you yourself see on the page. We suggest pausing and engaging the children in sharing their ideas when you are reading aloud the many open-ended questions in this book.

The book begins with an open-ended question phrased for teachers to ask: “What are some of the ways that people in your home help each other?” A parent might ask “What are some of the ways that we help each other in our family?” The first illustration suggests some ways that children might respond, for example, if they have experiences as family members with helping to care for a cat, cook dinner, sweep a floor, or make beds.

The next open-ended question is “What are some of the ways that you can think of to help your friends and neighbors?” This broadens the children's thinking beyond the family to the neighborhood. The process would be the same, to pause and invite the children to offer their insights to one another. The illustration portrays someone thinking nice thoughts about a neighbor while going to see if the neighbor might need some help in some way.

Then ask, “What are some of the jobs that people in our community do to help others?” This expands the view to essential workers and others in the children’s community. The use of “our” re-frames the conversation from the children’s own personal experiences (“your family” and “your neighborhood”) to a shared perspective of “our community.” The illustration is a collage of words representing many kinds of workers with whom the children may have interacted in the community.

Pause after asking such open-ended questions to give children time to think and to talk about their experiences helping and being helped by family members, friends and neighbors, and workers in the community. With a large group of children, teachers may need to give permission to speak, with a brief “please talk softly with your neighbor” and repetition of the question. Some teachers provide a name for this kind of whole group reading experience by teaching their students to “think/pair/share” when the teacher pauses after asking an open-ended question. (See, for example, <https://www.readingrockets.org/strategies/think-pair-share>)

Introducing the *Thank You Notes* theme

The next open-ended questions are “Who has been helpful to you? What would you write in a thank you note to that person?” These introduce the theme of writing thank you notes. The book’s protagonist, Jasmine, decides to write a thank you note each day and then also to ask a friend or family member to write one too. Each day, each note writer asks a sibling or a friend to write one, so that the number of thank you notes written grows exponentially, doubling each day.

The book provides examples of seven thank you notes written during the first three days of note writing: one on Sunday by Jasmine, two on Monday by Jasmine and her friend Makaila, and four on Tuesday by Jasmine, Makaila, Marcus, and Nadira. The text next describes eight more written during the fourth day of thank you note writing, Wednesday, by Jasmine, Makaila, Marcus, Nadira, Aliyah, Stephanie, Ray, and Ana.

Engaging children in thinking together about many aspects of exponential growth

Many questions throughout this book are open-ended; others are math problems solvable by using mathematics appropriate for children in grades 3-5: multiplying by 2 and adding multi-digit numbers as well as by drawing bar graphs. While you are reading this book with children, we encourage you to pause and engage the children in thinking together about questions such as those included in the text:

- How can Jasmine keep track of how many thank you notes have been written each day?
- What pattern do you see in the way the number of thank you notes increases each day?
- “If this pattern stays the same, how many thank you notes will we write on Thursday?” Jasmine wonders. “How many will we write on Friday? What about Saturday?”
- Marcus thinks, “I wonder what the number of thank you notes written each day looks like if I draw out the whole week?” How would you draw it?
- What is different about this one?
- How did you draw it?

- On Saturday, Makaila asks her Dad to please order envelopes for all the thank you notes. He asks, “How many do you need?”
- How many thank you notes will they write in the second week?
- What pattern will you use to fill in the number of thank you notes written each day for the second week?
- Marcus decides to make another drawing of the thank you notes he expects to be written during the second week. What does your drawing look like?
- Marcus wants his graph for the second week to fit on one page and seeks help from his older sister, Aliyah. She asks, “On your bar graph, what scale did you use?”
- What are some things you notice about the increase in thank you notes during the two weeks?
- How many envelopes will they need for the second week?
- Jasmine’s Mom is only willing to order 4 boxes of 100 envelopes each. Which day of the week will they run out of envelopes?
- How many people do you know who will write thank you notes if you ask for their help?

The appendix includes six additional mathematics questions and answers with which you can extend the thinking if the children express interest.

Encouraging children to write and illustrate their own stories

Before, during, or after reading this book, a teacher or parent may choose to engage the children in drawing pictures and writing their own stories about helping or being helped in family, neighborhood, and community-based ways. They also may be interested in writing their own thank you notes and creating colorful covers for their messages. Also appropriate would be drawing and writing stories about examples of exponential growth as well as making bar graphs to create their own visual images of this phenomenon.

The children may have had experiences making bar graphs at school by making vertical bars with post-it notes on a poster board or wall. The US Common Core State Mathematics Standards for Measurement and Data include learning how to make and interpret bar graphs in second grade and scaled graphs in third grade.

Making a bar graph representing this story could be memorable. Using small post-it notes (1.5” high x 2.0 “ wide), the children could start on the floor and make the vertical bars for each day on a wall by posting small post-it notes one above another for each day: one for Sunday (1.5” high at floor level), two post-it notes for Monday (3” high from floor level), four post-it notes for Tuesday (6” high from floor level), eight post-it notes for Wednesday (12” high floor level) from), sixteen post-it notes for Thursday (24” high from floor level), thirty-two post-it notes for Friday (48” high from floor level) and contemplate with wonder building a bar for Saturday with sixty-four post-it notes reaching up near the ceiling (96” high from floor level)!

Some children may have computer skills or older siblings or parents with knowledge of how to use free graph paper on a computer or computer graphing programs for making a graph for the predicted numbers of thank you notes to be written during the second week if the children in the book were to continue their writing thank you notes project.

Making connections to relevant national standards for mathematics, literacy, social studies, science, and visual arts

These activities engage children in exploring the concept of exponential growth by integrating mathematics, literacy, social studies, science and visual arts activities into a coherent series of lessons that meet many national standards for these curricular areas in grades 3-5.

Mathematics: The book invites children to participate in developing mathematical ways to represent exponential growth, for example, at a level matching the Common Core State Standards Initiative for Mathematics for grades 3-5 capabilities: to multiply by 2 (CCSS.MATH.CONTENT.4.NBT.5), to add multi-digit numbers (CCSS.MATH.CONTEXT.4.NBT.4), and to create bar graphs (CCSS.MATH.CONTENT.2.MD.D.10 and CCSS.MATH.CONTENT.3.MD.B.3). (<http://www.corestandards.org/Math/Content/3/MD/>)

Literacy: The book also models Common Core English Language Arts Standards in the *Speaking and Listening* strand such as CCSS.ELA-LITERACY.SL.4.1 to *engage effectively in a range of collaborative discussions...building on others' ideas and expressing their own clearly*. For example, Jasmine starts a table that lists the known number of thank you notes written on Sunday (1), Monday (2), Tuesday (4), and Wednesday (8). Then she wonders, "If this pattern stays the same, how many thank you notes will we write on Thursday?" Marcus and Aliyah discuss two different ways to figure this out. Jasmine later uses Aliyah's method for completing the table. The book also invites the reader to participate in the thinking during this process. In addition, the book models CCSS.ELA.W.4.10 in the *Writing* strand by showing an increasing group of children who are choosing to write daily for a purpose. (<http://www.corestandards.org/ELA-Literacy/>)

Social Studies: In addition, the book contributes to meeting social studies standards, particularly those relevant to civics such as the College, Career, and Civic Life (C3) Framework for Social Studies State Standards, D2.Civ.6.3-5, *Describe ways in which people benefit from and are challenged by working together, including through government, workplaces, voluntary organizations, and families*. For example, Makaila writes to her aunt who works in the police department, "Dear Auntie, 'Thank you for keeping our neighborhood safe so I can play in the park.' Kisses, Makaila." (<https://www.socialstudies.org/sites/default/files/2017/Jun/c3-framework-for-social-studies-rev0617.pdf> page 32).

Science: With respect to the *Next Generation Science Standards*, the book engages children in the science and engineering practice of a grades 3-5 aspect of ***developing and using models to describe and predict phenomena*** (NGSS Appendix F, p. 6). These include the table listing the number of thank you notes written for each day of the week. This book also explicitly engages children in the practice of ***using mathematics and computational thinking by organizing simple data sets to reveal patterns that suggest relationships and to describe, estimate, and graph quantities to address questions*** (NGSS Appendix F, p.10). For example, after the children complete the table for the first week, Marcus makes a bar graph to represent the numbers in the table.

The book also engages children in developing and using the NGSS crosscutting concept of *patterns*, specifically the grades 3-5 aspect that students *identify patterns related to time, including simple rates of change...and use these patterns to make predictions* (NGSS Appendix G, p. 4). For example, the children make a second table for the second week showing how many thank you letters they would need to write each day and use this to figure out how many envelopes they would need if they decide to keep on writing thank you notes for another week. (<https://www.nextgenscience.org/resources/ngss-appendices>)

Visual Arts: The illustrations include a variety of covers and texts for example thank you notes. These model the National Core Art Standards for Creating Visual Arts in grades 3-5 such as VA:Cr1.2.4a, *Collaboratively set goals and create artwork that is meaningful and has purpose to the makers.* (<https://www.nationalartsstandards.org>)

Exploring other contexts illustrating exponential growth

Some children's books present visual displays of exponential growth in stories that have a moral message. *One Grain of Rice: A Mathematical Folktale* by Demi, for example, illustrates what happens when a young girl in India tricks a greedy ruler into feeding his people by asking for a reward of a grain of rice that doubles each day for 30 days. Pictures and a calendar show how many grains are needed with 536,870,912 on Day 30 and a total of more than a billion. *The 512 Ants on Sullivan Street* by Carol Losi provides a comical image of exponential growth with a chant about what 1, 2, 4, 8, 16, 32, 64, 128, 256, and 512 ants carry from a basket packed for a picnic. Both of these books illustrate in a vivid way the slow start that builds into an overwhelming number of grains of rice or ants that is characteristic of exponential growth.

A visual image of the power of exponential growth in the context of compound interest is provided by a financial literacy course at Pennsylvania State University. This resource also develops the mathematical formula for exponential growth in ways appropriate for interested older students and adults (see <https://psu.instructure.com/courses/1806581/pages/chapter-1-basic-concepts-compound-interest-and-time-value-of-money>).

In *The Limits of Growth*, Donella Meadows, Dennis Meadows, Jørgen Rander, and William Behrens described a famous French riddle about the limits of exponential growth in the context of water lilies growing in a pond (p. 29). The back cover of *Thank You Notes* includes six images representing this riddle. If there is one water lily pad on a pond, the number of water lily pads doubles each day, and the water lily pads cover the pond in 30 days, on what day must one clear the pond if one waits until half of the pond is covered? (Day 29). One way to make this riddle intriguing to children is to add a context – a family rents a cabin near a pond, agrees to keep the pond clear of water lilies, enjoys the pond daily but not much seems to be happening so they decide to take a few days to sightsee in the area. What does the pond look like if they don't leave until Day 27? How soon must they be back to clear half the pond of water lilies?

These examples of exponential growth may inspire conversations about the impact of the increasing number of people on the Earth. In a local context, for example, what would happen if a new restaurant opens with recyclable take-out containers, the number of customers each week keeps doubling, and these customers throw away their containers in the trash rather than

recycling them? What would happen if the number of cars on the roads increases exponentially? What will happen if the Earth's average temperature keeps getting hotter and hotter? An open source resource is available for communicating with children, teachers, the general public, and policy makers about evidence for the Earth's increasing global temperature and its impact on climate change. See: <https://open.oregonstate.education/physicsforteachers/chapter/considering-the-evidence-for-global-climate-change/> .

Children also may recognize the exponential-like pattern of graphs included in news about the COVID-19 pandemic. These graphs can ground a conversation about both exponential growth and decay when they show both the steep incline of the curve representing increasing number of cases and the 'flattening' of the curve that occurs with use of the recommended actions of wearing masks, social distancing, and getting vaccinated. An example based on world wide data compiled by the World Health Organization is informative (see: <https://worldhealthorg.shinyapps.io/covid/>). Older students, teachers and parents may find interesting a report by the US National Academy of Sciences about improving the public's understanding of the surprisingly steep increase in coronavirus cases when infections are increasing exponentially (see: <https://www.pnas.org/content/117/28/16264>).

Teachers and parents will need to choose if, when, and how to discuss such topics with their children. Making a connection to what is happening during the COVID-19 pandemic can help children understand what happens if one person infects others, if those each infect others, and if the number who get sick exceed the number of available hospital beds. The exploration of the concept of exponential growth in this book may help children understand better why their school had closed, why they had been not going to places they used to enjoy, and why they had to miss the group activities that once had filled their days.

Outline of *Thank You Notes: A Children's Story about Exponential Growth*

Page 1: Cover with title, authors, and bar graph made of covers of thank you notes for first 4 days: (1, 2, 4, 8).

Page 2: Publication information.

Pages 3, 4, 5: Questions posed about helping others:

What are some of the ways that people in your home help each other?

What are some of the ways that you can think of to help your friends and neighbors?

What are some of the jobs that people in our community do to help others?

Page 6, 7: Protagonist, Jasmine, decides to write thank you notes.

Page 8: On Sunday, Jasmine writes her first thank you note, to her sister.

Page 9: Questions posed: Who has been helpful to you?

What would you write in a thank you note to that person?

Pages 10-15: On Monday, Jasmine invites a friend to join her in writing thank you notes. Each day each note writer invites someone to join the thank you note writing process: Sunday (1 note writer), Monday (2 note writers), Tuesday (4 note writers), Wednesday (8 note writers).

Pages 16-17: Question posed: How can Jasmine keep track of how many thank you notes have been written each day?

Pages 18-19: Table provided, Sunday through Saturday, with numbers posted for Sunday through Wednesday (1, 2, 4, 8). Question posed: What pattern do you see in the way the number of thank you notes increases each day?

Pages 20-23: Two thank you note writers discuss the patterns they see. One sees that the number for the next day equals the number for the previous day added to itself. The other sees that the number for the next day equals two times the number for the previous day.

Page 24-25: Questions posed: How many thank you notes will be written on Thursday? Friday? Saturday?

Marcus wonders what the numbers written each day would look like if drawn for the whole week. Question posed: How would you draw it?

Page 26-27: "There are many ways to draw it" with example shown of Marcus's bar graph with vertical bars.

Page 28-29: Example of bar graph with horizontal bars. Questions posed: What is different about this one? What does your drawing look like?

Page 29: Table shown with numbers for all seven days of the first week.

Page 30: Calculation shown for finding out how many envelopes are needed, $1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$ envelopes.

Pages 31-33: Two of the children discuss whether to continue writing thank you notes, with each note writer inviting someone to join their project each day. Question posed: How many thank you notes will they write in the second week?

Page 34-35: Table for first week shown with numbers 1, 2, 4, 8, 16, 32, 64. Table for second week shown with Sunday's number provided, 128, the rest of the week's numbers are blank. Question posed: What pattern will you use to fill in the number of thank you notes written each day for the second week?

Page 36: Table shown with numbers for second week filled in (128, 256, 512, 1024, 2048, 4096, 8,192). Marcus decides to draw a graph for the second week. Question posed: How would you make one?

Page 37: Marcus wants his graph for the second week to fit on one page and seeks help from his older sister, Aliyah. She asks, "On your bar graph, what scale did you use?"

Page 38-39: Marcus had used one unit to represent one thank you note. Bar graph shown.

Page 40-41: Marcus tries one unit representing 10 thank you notes. Only up to 900 thank you notes would fit on one page. Bar graph for the first day of the second week (128 thank you notes) shown.

Page 42-43: Marcus tries one unit representing 50 thank you notes. Only up to 4,500 thank you notes would fit on one page. Bar graph for the first day of the second week (128 thank you notes) shown.

Page 44-45: Marcus tries one unit representing 100 thank you notes. It works! Up to 9,000 thank you notes would fit on one page so Saturday's 8,192 thank you notes fit! Bar graph is shown on one page for all the thank you notes written during the second week.

Page 46-47: Marcus draws a bar graph for both weeks 1 and 2. Question posed: What are some things you notice about the increase of thank you notes during the two weeks?

Page 48-49: Discussion of ordering envelopes for the second week. Question posed: How many envelopes will they need for the second week?

Page 50-51: Table with numbers filled out for second week (128, 256, 512, 1024, 2048, 4096, 8192) and calculation shown for finding out how many envelopes are needed: 16,256! Parent refuses to order that many envelopes; instead parent offers to order only 4 boxes of 100 each. Questions posed:
Which day of the week will they run out of envelopes?

How many people do you know who will write thank you notes if you ask for their help?

Page 52-53: Appendix: Six additional mathematics questions to ponder, with answers provided.

Page 54: Another way of thinking about exponential growth, based on the book that inspired this children's book, *The Limits of Growth* by Meadows, Meadows, Randers, and Behrens, published in 1972.

Page 55: About the authors.

Text of *Thank You Notes: A Children's Story about Exponential Growth*

- p. 1 Cover: *Thank You Notes: A Children's Story about Exponential Growth*
[Illustration: bar graph made of 1, 2, 4, 8 thank you note covers]
- p. 2 [Publication information]
- p. 3 What are some of the ways that people in your home help each other?
[Illustration: Collage picturing a cat with food dish, a pan on stove, dustpan and broom, teddy bear in made bed]
- p. 4 What are some of the ways that you can think of to help your friends and neighbors?
[Illustration: Someone thinking nice thoughts about a neighbor while going to see if the neighbor might need some help in some way.]
- p. 5 What are some of the jobs that people in our community do to help others?
[Illustration: Collage of words: Firefighter, Bus Driver, Health Care Worker, Teacher, Police Officer, Grocery Store Worker, Garbage Collector, Power Plant Worker]
- p. 6-7 After thinking about all the ways that people in our community are helping each other, Jasmine decides she wants to write thank you notes.
[Illustration: Thank you note's cover with large letters spelling THANK You in rainbow colors]
- p. 8 On Sunday evening, Jasmine writes the first thank you note to her sister who showed her the moon while they were taking out the garbage.
[Illustration: Small rectangle: Calendar Sunday; also thank you note cover (two children looking at a crescent moon) and content of example thank you note (Dear sister, Thank you for showing me the moon. You make talking out the garbage more fun! With love, Jasmine.)]
- p. 9 Who has been helpful to you? What would you write in a thank you note to that person?
[Illustration: Three large question marks spaced far apart]
- p. 10 On Monday, Jasmine writes a second thank you note and asks her best friend Makaila to help. Jasmine writes to her uncle who is a nurse.
[Illustration: Small rectangle: Calendar Monday; also example thank you note with cover (two red flowers) and content (Dear Uncle, Thank you for thinking of my health, and talking care of sick people in our community. With Love, Jasmine.)]
- p. 11 Makaila writes to her mom who made her a picnic lunch to enjoy in the park.
[Illustration: Example thank you note with cover (three yellow flowers) and content (Dear Mom, Thank you for making me lunch! I enjoyed our trip to the park. I love the flowers. Love, Makaila)]

- p. 12 On Tuesday, Jasmine and Makaila each write one thank you note and each asks another person to help write more thank you notes.
 Jasmine asks her younger brother Marcus.
 Makaila asks her older sister Nadira.
 Jasmine writes a thank you note to her teacher about their field trip to the playground to observe clouds in the sky.
 [Illustration: Small rectangle: Calendar Tuesday and example thank you note cover (blue sky and clouds)]
- p. 13 Makaila writes to her aunt who works in the police department.
 Marcus writes to his mom who helped him with his homework.
 Nadira writes to her dad who helped her clean out the cat's litter box.
 [Illustrations: Four example thank you notes with content:
 (Dear Miss Garcia, I love the shapes different clouds have! Thank you for taking our class outside to look at them! Your student, Jasmine)
 (Dear Auntie, Thank you for keeping our neighborhood safe so I can play in the park. Kisses, Makaila)
 (Hi Mom, Thanks for going over my homework with me. I like hearing your stories about when you were my age. Marcus)
 (Dear Dad, My cat is so cuddly, but her litter box smells bad. Thank you for helping me clean it out. Love, Nadira)]
- p. 14 On Wednesday Jasmine, Makaila, Marcus, and Nadira each write one thank you note and each asks another person to help write more thank you notes.
 Jasmine asks her older sister Aliyah.
 Makaila asks her friend Stephanie.
 Marcus asks his friend Ray.
 Nadira asks her friend Ana.
 [Illustration: Small rectangle: Calendar Wednesday]
- p. 15 Jasmine writes a thank you note to the mail carrier. Makaila writes to her neighbor who works at the grocery store.
 Marcus writes to his uncle who is a firefighter. Nadira writes to her cousin who drives a bus.
 Aliyah writes to Jasmine who helped her cook dinner. Stephanie writes to her dad who helped her do the laundry. While it was in the dryer, they went outside to enjoy the sunset.
 Ray writes to his mom who works at the power plant. Ana writes to her sister who played a video game with her even though she didn't really want to.
 [Illustrations: Covers of 8 thank you notes (Sunburst; Thank You; three flowers; sun and a flower; seashore in moon light; sunrise; three flowers; seashore at sunset)]
- p. 16 *This is a lot of thank you notes to keep track of, Jasmine thought. I want to keep writing thank you notes for one whole week.*
 [Illustration: Table with one row and seven boxes labeled for the days of the week]

p. 17 How can Jasmine keep track of how many thank you notes have been written each day?

p. 18-19 There are lots of different ways to keep track.

Jasmine decides, *I will write the days of the week and the number of thank you notes we wrote that day.*

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	
Friday	
Saturday	

What pattern do you see in the way the number of thank you notes increases each day?

p. 20-21 There are many ways to describe this pattern.

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	
Friday	
Saturday	

Marcus did it like this:

On Monday we wrote 2 thank you notes, that was _____ + _____
(fill in the blanks with the number of notes written on Sunday)

On Tuesday we wrote 4 thank you notes, that was _____ + _____
(fill in the blanks with the number of notes written on Monday)

On Wednesday we wrote 8 thank you notes, that was _____ + _____
(fill in the blanks with the number of notes written on Tuesday)

Marcus says to Aliyah, “Look, I notice that each day the number of thank you notes we write is the number we wrote the day before added to itself.”

p. 22-23

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	
Friday	
Saturday	

Aliyah did it like this:

On Monday we wrote 2 thank you notes,
that was _____ x the number written on Sunday

On Tuesday we wrote 4 thank you notes,
that was _____ x the number written on Monday

On Wednesday we wrote 8 thank you notes,
that was _____ x the number written on Tuesday

Aliyah says to Marcus, “Your way is interesting. I notice that if we multiply the number of thank you notes written the day before by 2, it equals the number we write today.”

Jasmine giggles “That’s called doubling!!!”

p. 24-25 “If this pattern stays the same, how many thank you notes will we write on Thursday?”
Jasmine wonders.

“How many will we write on Friday?”

“What about Saturday?”

p. 26-27 Marcus thinks, *I wonder what the number of thank you notes written each day looks like if I draw out the whole week.*

How would you draw it?

There are many ways of drawing it. Marcus did it like this as a bar graph:

[Illustration: Graph with vertical bars for the first seven days of the thank you note project]

p. 28-29 Marcus could have drawn it like this instead:

[Illustration: Graph with horizontal bars for the first seven days of the thank you note project]

What is different about this one?
What does your drawing look like?

On Saturday, Makaila asks her Dad to please order envelopes for all the thank you notes. He asks, “How many do you need?” She adds up all the thank you notes written that week.

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	16
Friday	32
Saturday	64

p. 30 We wrote $1+2+4+8+16+32+64 = 127$ thank you notes in one week!
Makaila’s Dad orders 127 envelopes for the thank you notes that were written from Sunday through Saturday.
[Illustration: Array of rainbow-colored envelopes]

p. 31 Makaila calls Jasmine and asks if she thinks they should write more thank you notes for one more week.

“Let’s do it,” Jasmine agrees, “If we keep asking someone new to help each day like last week, we can get a lot of thank you notes written!!!”

“Oh, my mom says ask your dad to send her the link to the envelopes he ordered, and she will order the same ones for next week.”

How many thank you notes do you think they will write in the second week?

p. 32-33 They plan to keep the same pattern of each person writing one thank you note every day and asking a new person each day to also write a thank you note.

Jasmine dances around giggling “Doubling, doubling, doubling...!”
[Illustration: big question mark surrounded by small boxes, each with $2x_ _$ - inside]

p. 34-35 Jasmine starts writing in the number of thank you notes she expects to be written during the second week, if they keep the same pattern from the first week.

For Sunday she multiplies the number of notes written the day before, 64, by 2.
She uses the pattern her sister Aliyah found from the first week.

What pattern will you use to fill in the number of thank you notes written each day for the second week?

First Week

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	16
Friday	32
Saturday	64

Second Week

Days	N
Sunday	128
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

- p. 36-37 Marcus decides to make another drawing of the thank you notes he expects to be written during the second week.

How would you make one?

Marcus encounters a problem, however.

“Aliyah, can you help me? The second week doesn’t fit on my graph paper. I already tried taping five pieces of graph paper together and that’s still not big enough.”

Aliyah ask, “On your bar graph, what scale did you use?”

- p. 38-39 “Last time I made my scale to have one unit equal to one thank you note.”
[Illustration: Bar graph of Number of Thank You Notes Written for the first week.]

“Marcus, if you want the second week to all fit on one piece of graph paper, one unit will have to represent more than one thank you note.”

- p. 40-41 First Marcus tries one unit equal to 10 thank you notes.
When he draws it on his graph paper, this scale only fits up to 900 thank you notes.
Not enough. On the last day of week two, if the pattern stays the same, 8,192 thank you notes will be written!
[Illustration: Bar graph with scale of one unit representing 10 thank you notes.]

p. 42-43 Next he tries one unit equal to 50 thank you notes.
 When he draws it on his graph paper, this scale only fits up to 4,500 thank you notes.
 Still not enough.
 [Illustration: Bar graph with scale of one unit representing 50 thank you notes.]

p. 44-45 Finally, Marcus tries one unit equal to 100 thank you notes.
 It worked! This scale fits 9,000 thank you notes on one piece of graph paper.
 This is how he drew it.
 [[Illustration: Bar graph with scale of one unit representing 100 thank you notes.]
 “Wow! That’s A LOT of thank you notes!!! I’m going to draw both weeks on one
 graph to see what it looks like.”

p. 46-47 [Illustration: Graph with vertical bars projected for two weeks of thank you note
 writing]
 What are some things you notice about the increase of thank you notes during the two
 weeks?

p. 48-49 “Mom!” calls Jasmine, “Did you order the envelopes for next week yet?”

“I’m doing it right now. Makaila’s dad said he ordered 127 envelopes for last week. Is
 that how many you want for next week?”

“We’ll need a lot more than that; I’ll just add it up now.”

How many envelopes will they need for the second week?

Second Week

Days	N
Sunday	128
Monday	256
Tuesday	512
Wednesday	1024
Thursday	2048
Friday	4096
Saturday	8192

p. 50-51 $128+256+512+1,024+2,048+4,096+8,192 = 16,256$ envelopes!!!

“Mom, can you please order 16,256 envelopes?” asks Jasmine.

“What? That is way too many envelopes!! The link Makaila’s dad sent me lists boxes
 of 100 envelopes. I can get you 4 boxes, but that’s it.”

Which day of the week will they run out of envelopes?

How many people do you know who will write thank you notes if you ask for their help?

p. 52-53 Appendix: Additional questions to ponder...

Which day of the second week had almost the same amount of thank you notes as the whole first week?

Sunday

*In the whole first **week** 127 thank you notes are written; on the first **day** of the second week, 128 thank you notes are written.*

How many more envelopes did Jasmine's mom order than Makaila's dad?

273 more envelopes

4 x boxes of 100 envelopes = 400 envelopes. Makaila's dad ordered 127 envelopes.

400 - 127 = 273

Makaila's dad's envelopes were enough for 7 days in the first week. How many days did Jasmine's mom's envelopes last in the second week?

2 full days

128 + 256 = 384 thank you notes written on Sunday and Monday of the second week with only 400-384 = 16 available for Tuesday

How many people will Jasmine have asked if she asks a new person to help her write thank you notes each day during the two-week period?

13 people

How many people did Jasmine's brother Marcus ask to help write thank you notes during the two-week period? Marcus started writing notes on Tuesday.

11 people

How many thank you notes could you and the people you asked to help write, if you all followed the same pattern of each person writing one thank you note and asking a new person to write one thank you note each day?

p. 54 This story was inspired by a book, *The Limits of Growth*. The authors included a French riddle to illustrate their topic, the initially slow and then rapid nature of exponential growth that can overwhelm limited resources. Here is a short version of the riddle: On Day 1, there is one lily pad in a pond. The number of lily pads doubles every day. If unchecked, the lily pads will cover the pond in 30 days, killing other life in the pond. If you decide to wait to clear the lily pads until they cover half the pond, what day will that be?

[Illustration: six versions of water lily pads in a pond]

A common answer is half-way to the end of the month, but on Day 15 the area covered is still barely noticeable. If you wait until the lily pads cover half of the pond, you will be frantically clearing the pond on Day 29! Meadows, D. H., D. L. Meadows, J.

Randers, and W. W. Behrens, III. 1972. *The Limits to Growth*. New York: Universe Books, p. 29.

For a visual model of the lily pond riddle, see:

<https://www.youtube.com/watch?v=0BSaMH4hINY> About the Authors

p. 55

About the Authors

We wrote this book together while in quarantine on opposite coasts of the United States during the beginning of the COVID-19 pandemic. We had discussions on Zoom after remote sessions for a lab-based physics course for prospective elementary teachers that Emily was teaching. (see: open.oregonstate.edu/physicsforteachers)An associate professor of science education at Oregon State University, Emily has long been an advocate for climate change education and has a fondness dating back years for the lily pad example of exponential growth and its limits. Katrina remembered this and started writing the story in a context reflecting her own teaching setting.

Seeing a pattern of exponential growth in the infection rate for Covid-19 was disquieting, especially for Katrina who lives in the greater New York City area. Prior to the coronavirus shutdown, Katrina, an informal science educator at Liberty Science Center, was teaching elementary students in a partnership school in an area that was one of the first clusters of Covid-19 infections in New Jersey. In thinking of these students and how to explore the concept of exponential growth in the context of this global pandemic, Katrina thought of using thank you notes to family, friends, neighbors, and essential workers as the vehicle for fostering understanding of the phenomenon of exponential growth.

The opening questions in the book: “What are some of the ways that people in your home help each other? What are some of the ways that you can think of to help your friends and neighbors? What are some of the jobs that people in our community do to help others?”, are how Katrina would start a class discussion on this topic. The discussion begins with a story that is personal to each student, broadens to include friends and neighbors, and then expands to the essential workers and the whole community.

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