Thank You Notes:
A Children's Story
about
Exponential Growth

By Katrina van Zee and Emily van Zee

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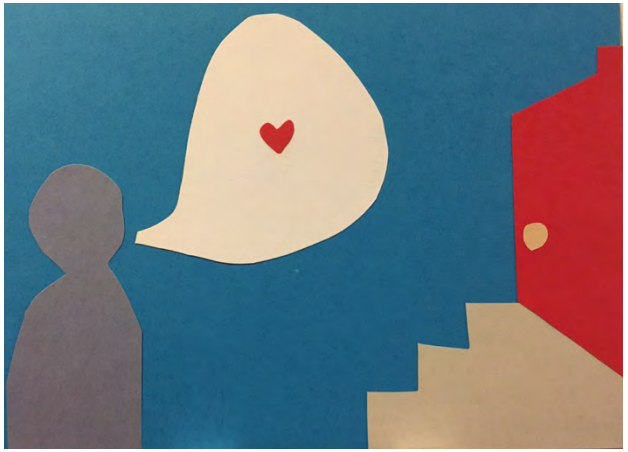
Artist: Katrina van Zee

2020

What are some of the ways that people in your home help each other?

What are some of the ways that you can think of to help your friends and neighbors?



What are some of the jobs that people in our community do to help others?

Firefighter

Bus Driver

Teacher

Health Care Worker

Police Officer

Grocery Store Worker

Garbage Collector

Power Plant Worker

After thinking about all the ways that people in our community are helping each other, Jasmine decides she wants to write thank you notes.

Calendar
Sunday

On Sunday evening, Jasmine writes the first thank you note to her sister who showed her the moon while they were taking out the garbage.

Who has been helpful to you? What would you write in a thank you note to that person?

?

?

?

Calendar
Monday

On Monday, Jasmine writes a second thank you note and asks her best friend Makaila to help. Jasmine writes to her uncle who is a nurse.

Makaila writes to her mom who made her a picnic lunch to enjoy in the park.

Calendar
Tuesday

On Tuesday, Jasmine and Makaila each write one thank you note and each asks another person to help write more thank you notes.

Jasmine asks her younger brother Marcus.

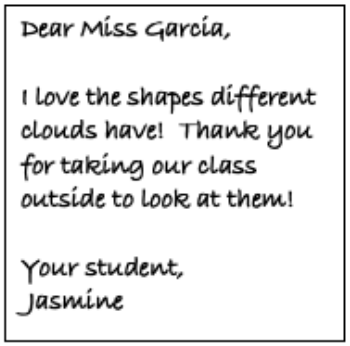
Makaila asks her older sister Nadira.

Jasmine writes a thank you note to her teacher about their field trip to the playground to observe clouds in the sky.

Makaila writes to her aunt who works in the police department.

Marcus writes to his mom who helped him with his homework.

Nadira writes to her dad who helped her clean out the cat's litter box.


Dear Miss Garcia,

I love the shapes different clouds have! Thank you for taking our class outside to look at them!

Your student,
Jasmine


Dear Auntie,

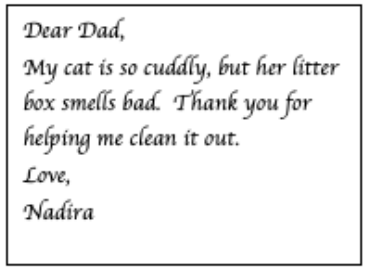
Thank you for keeping our neighborhood safe so I can play in the park.

Kisses,
Makaila


Hi Mom,

Thanks for going over my homework with me. I like hearing your stories about when you were my age.

Marcus


Dear Dad,
My cat is so cuddly, but her litter box smells bad. Thank you for helping me clean it out.
Love,
Nadira

Calendar
Wednesday

On Wednesday, Jasmine, Makaila, Marcus, and Nadira each write one thank you note and each asks another person to help write more thank you notes.

Jasmine asks her older sister Aliyah.

Makaila asks her friend Stephanie.

Marcus asks his friend Ray.

Nadira asks her friend Ana.

Jasmine writes a thank you note to the mail carrier. Makaila writes to her neighbor who works at the grocery store.

Marcus writes to his uncle who is a firefighter. Nadira writes to her cousin who drives a bus.

Aliyah writes to Jasmine who helped her cook dinner.

Stephanie writes to her dad who helped her do the laundry. While it was in the dryer, they went outside to enjoy the sunset.

Ray writes to his mom who works at the power plant. Ana writes to her sister who played a video game with her even though she didn't really want to.

This is a lot of thank you notes to keep track of Jasmine thought. *I want to keep writing thank you notes for one whole week.*

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

How can Jasmine keep track of how many thank you notes have been written each day?

There are lots of different ways to keep track.

Jasmine decides, *I will write the days of the week and the number of thank you notes we wrote that day.*

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	
Friday	
Saturday	

What pattern do you see in the way the number of thank you notes increases each day?

There are many ways to describe this pattern.

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	
Friday	
Saturday	

Marcus did it like this:

On Monday we wrote 2 thank you notes, that was ___ + ___
(fill in the blanks with the number of notes written on Sunday)

On Tuesday we wrote 4 thank you notes, that was ___ + ___
(fill in the blanks with number of notes written on Monday)

On Wednesday we wrote 8 thank you notes, that was __ + __
(fill in the blanks with the number of notes written Tuesday)

Marcus says to Aliyah, “Look, I notice that each day the number of thank you notes we write is the number we wrote the day before added to itself.”

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	
Friday	
Saturday	

Aliyah did it like this:

On Monday we wrote 2 thank you notes,
that was _____ x the number written on Sunday

On Tuesday we wrote 4 thank you notes,
that was _____ x the number written on Monday

On Wednesday we wrote 8 thank you notes,
that was _____ x the number written on Tuesday

Aliyah answers Marcus, "That's interesting. I notice that if we multiply the number of thank you notes written the day before by 2, it equals the number we write today."

Jasmine giggles "That's called doubling!!!"

"If this pattern stays the same, how many thank you notes will we write on Thursday?" Jasmine wonders.

"How many will we write on Friday?"

"What about Saturday?"

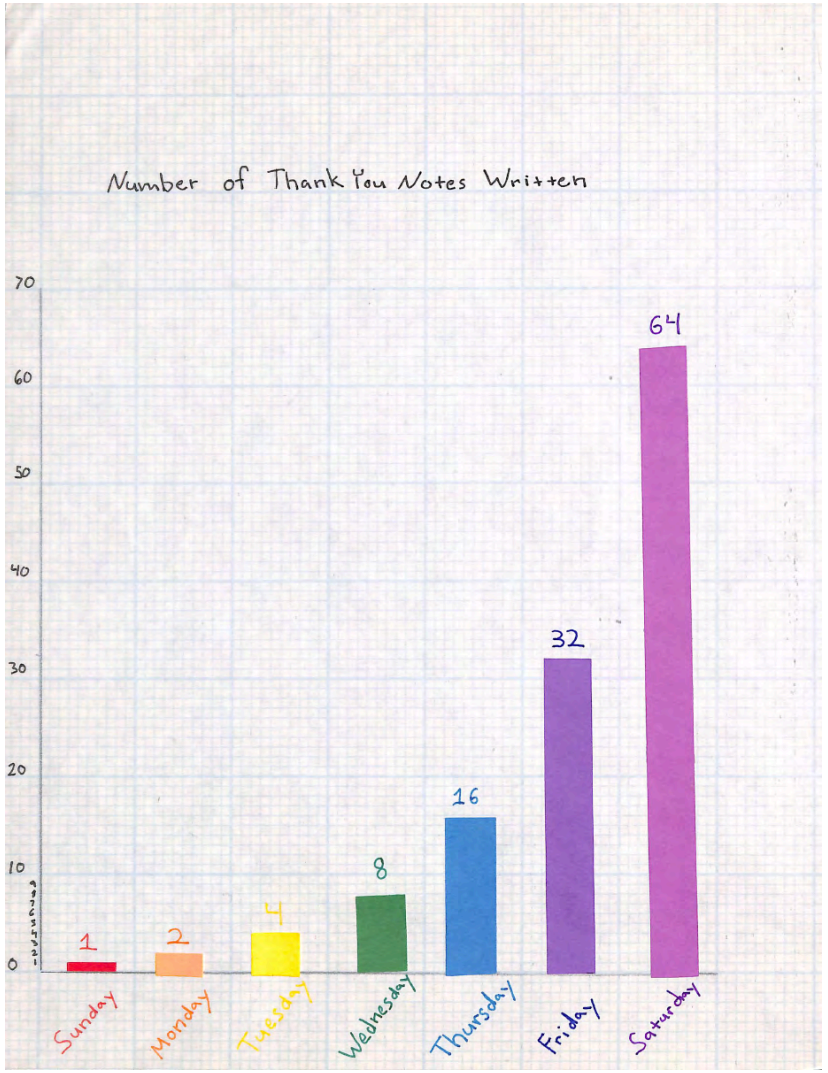
Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	
Friday	
Saturday	

Marcus thinks, *I wonder what the number of thank you notes written each day looks like if I draw out the whole week*.

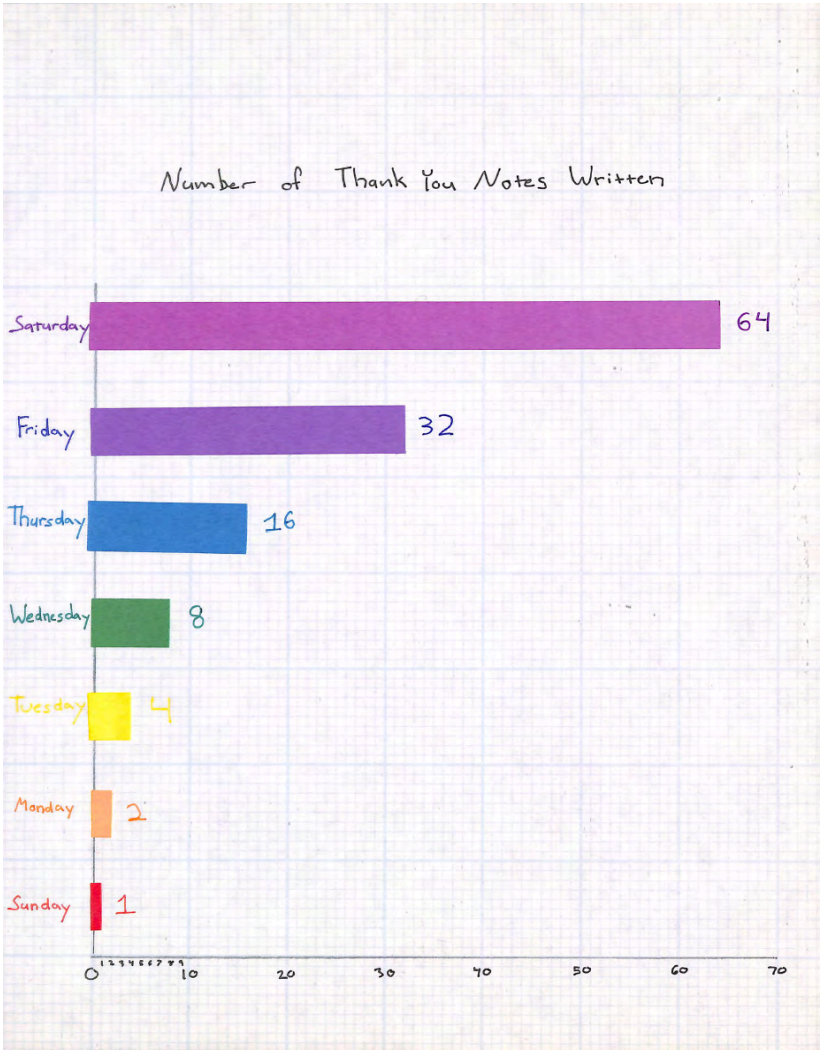
Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	16
Friday	32
Saturday	64

How would you draw it?

There are many ways of drawing it. Marcus did it as a bar graph.



Marcus could have drawn it like this instead:



What is different about this one?

What does your drawing look like?

On Saturday, Makaila asks her Dad to please order envelopes for all the thank you notes.

He asks, “How many do you need?”

She adds up all the thank you notes written that week.

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	16
Friday	32
Saturday	64

"We wrote 1+2+4+8+16+32+64 = 127 thank you notes in one week!"

Makaila's Dad orders 127 envelopes for the thank you notes that were written from Sunday through Saturday.

Makaila calls Jasmine and asks if she thinks they should write more thank you notes for one more week.

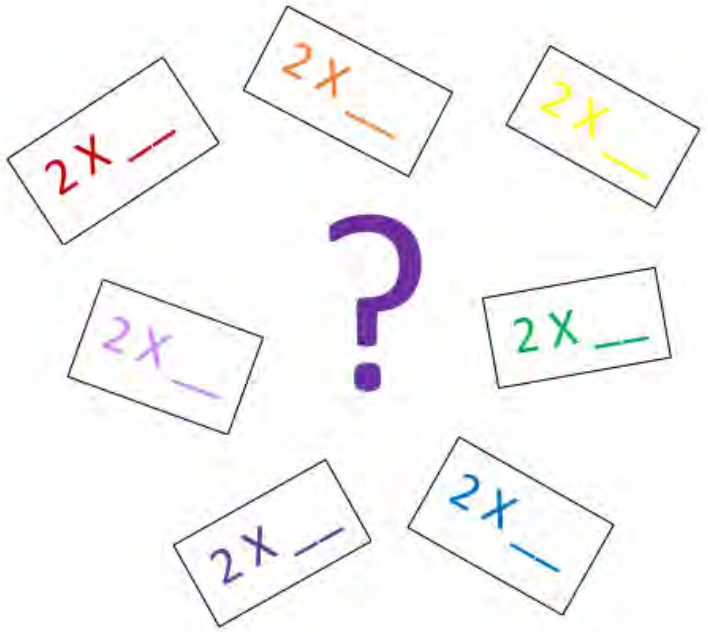
"Let's do it" Jasmine agrees, "If we keep asking someone new to help each day like last week, we can get a lot of thank you notes written!!!"

"Oh, my mom says ask your dad to send her the link to the envelopes he ordered, and she will order the same ones for next week."

How many thank you notes will they write in the second week?

They plan to keep the same pattern of each person writing one thank you note every day and asking a new person each day to also write a thank you note.

Jasmine dances around giggling “Doubling, doubling, doubling…!”



Jasmine starts writing in the number of thank you notes she expects to be written during the second week, if they keep the same pattern from the first week.

First Week

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	16
Friday	32
Saturday	64

Second Week

Days	N
Sunday	128
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	

For Sunday she multiplies the number of notes written the day before, 64, by 2.

She uses the pattern her sister Aliyah found in the first week.

What pattern will you use to fill in the number of thank you notes written each day for the second week?

First Week

Days	N
Sunday	1
Monday	2
Tuesday	4
Wednesday	8
Thursday	16
Friday	32
Saturday	64

Second Week

Days	N
Sunday	128
Monday	256
Tuesday	512
Wednesday	1024
Thursday	2048
Friday	4096
Saturday	8192

Marcus decides to make a graph of the thank you notes he expects to be written during the second week.

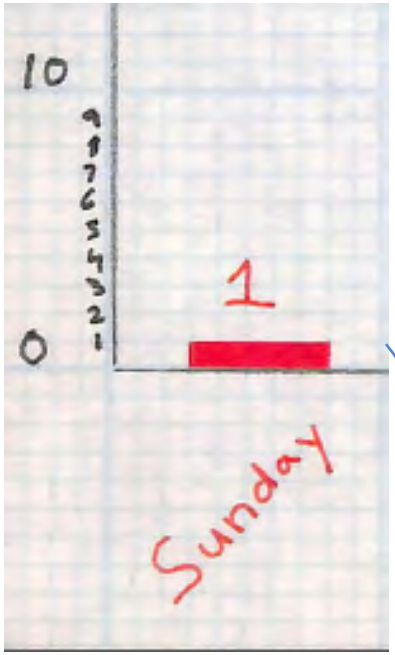
How would you make one?

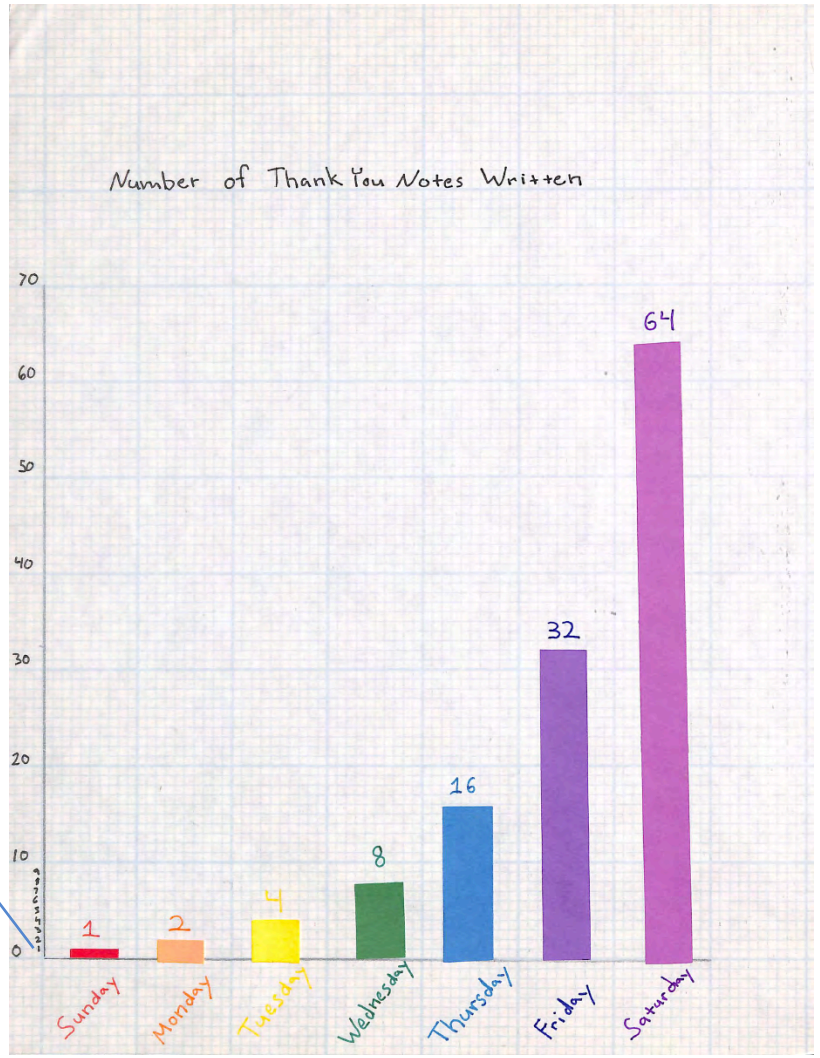
Marcus encounters a problem, however.

"Aliyah, can you help me? The second week doesn't fit on my graph paper. I already tried taping five pieces of graph paper together and that's still not big enough."

Aliyah asks, "On your bar graph, what scale did you use?"

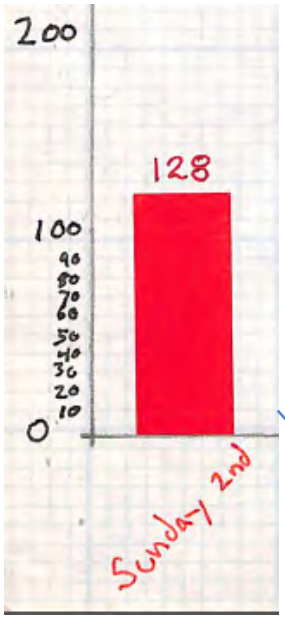
"Last time I made my scale to have one unit equal to one thank you note."





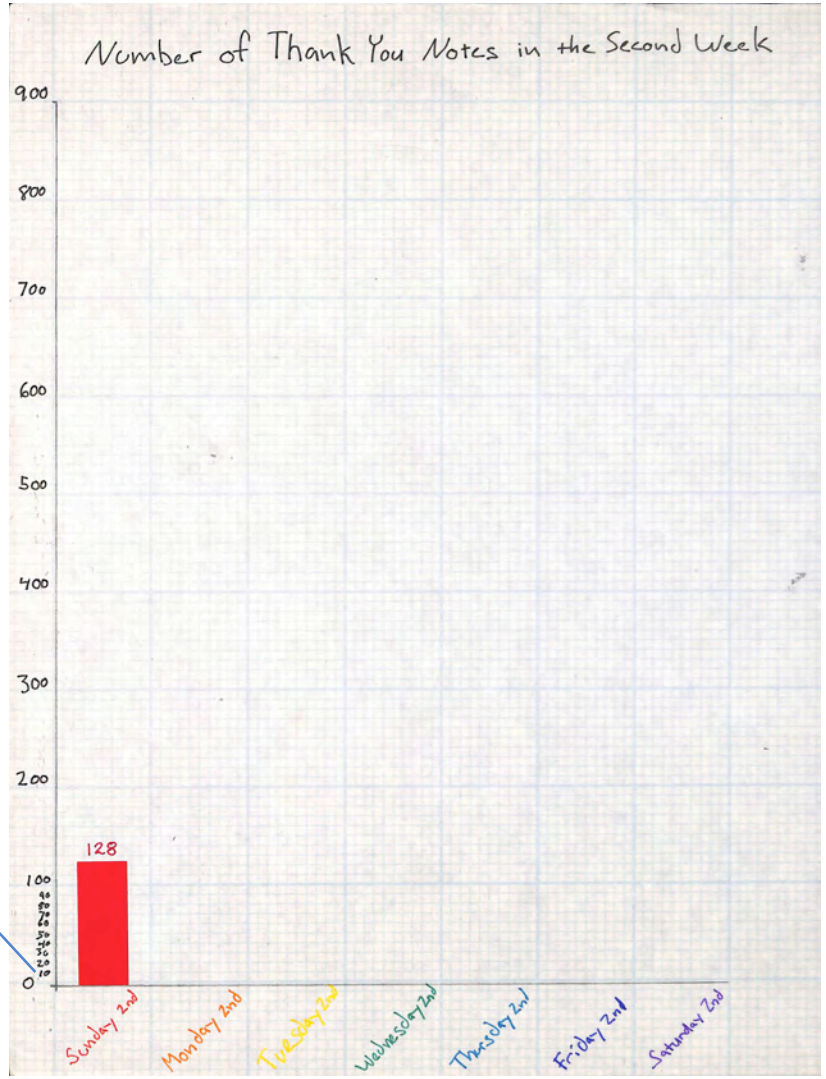
"Marcus, if you want the second week to all fit on one piece of graph paper, one unit will have to represent more than one thank you note."

First Marcus tries one unit equal to 10 thank you notes.

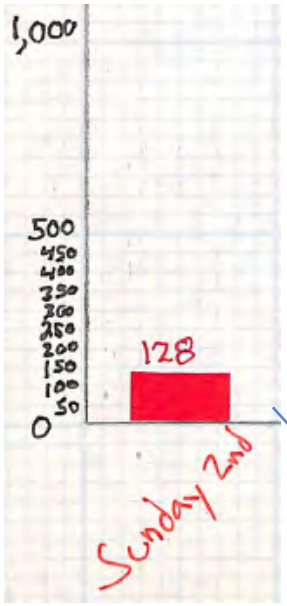


When he draws it on his graph paper, this scale only fits up to 900 thank you notes.

Not enough. On the last day of week two, if the pattern stays the same, 8,192 thank you notes will be written!

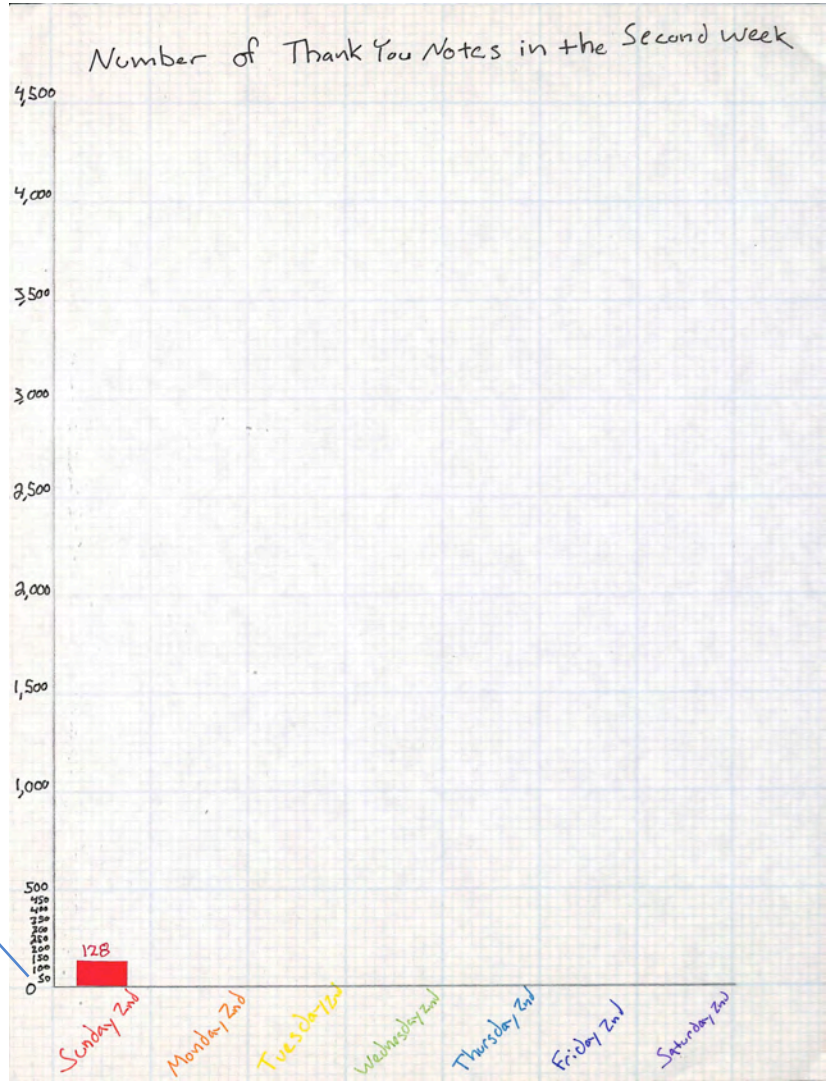


Next he tries one unit equal to 50 thank you notes.

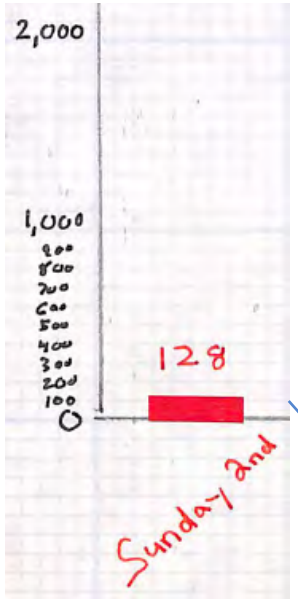


When he draws it on his graph paper, this scale only fits up to 4,500 thank you notes.

Still not enough.



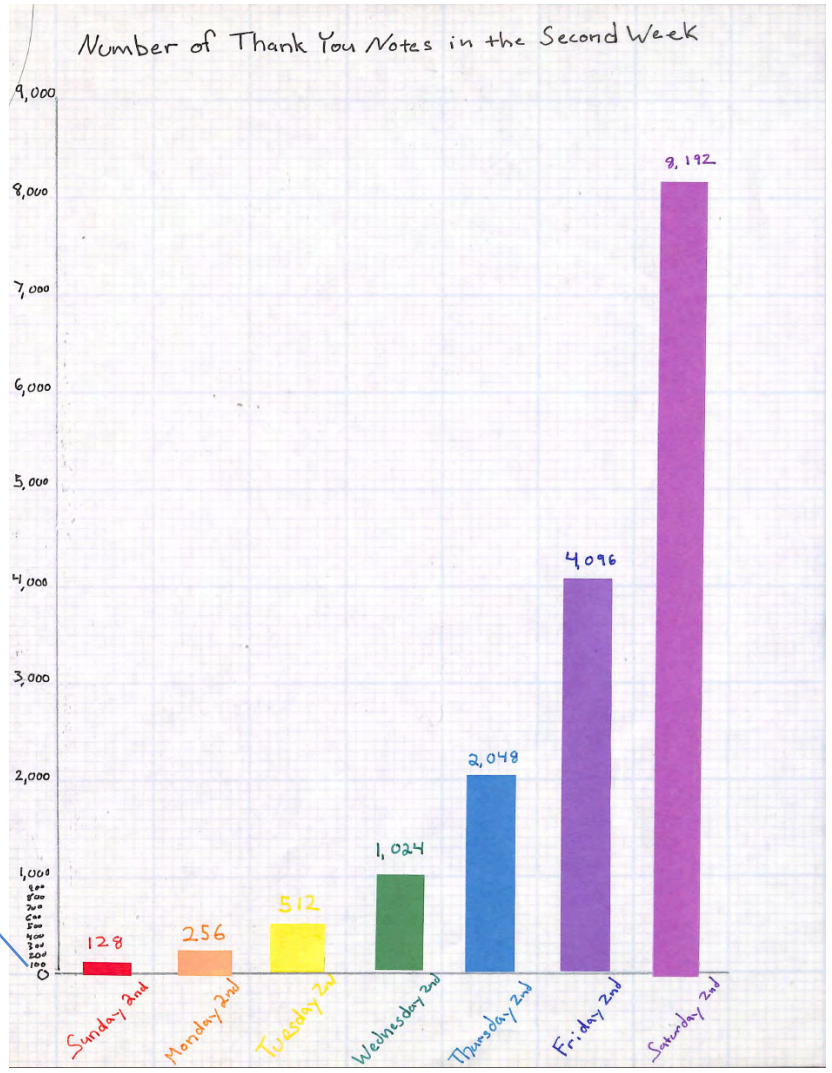
Finally, Marcus tries one unit equal to 100 thank you notes.



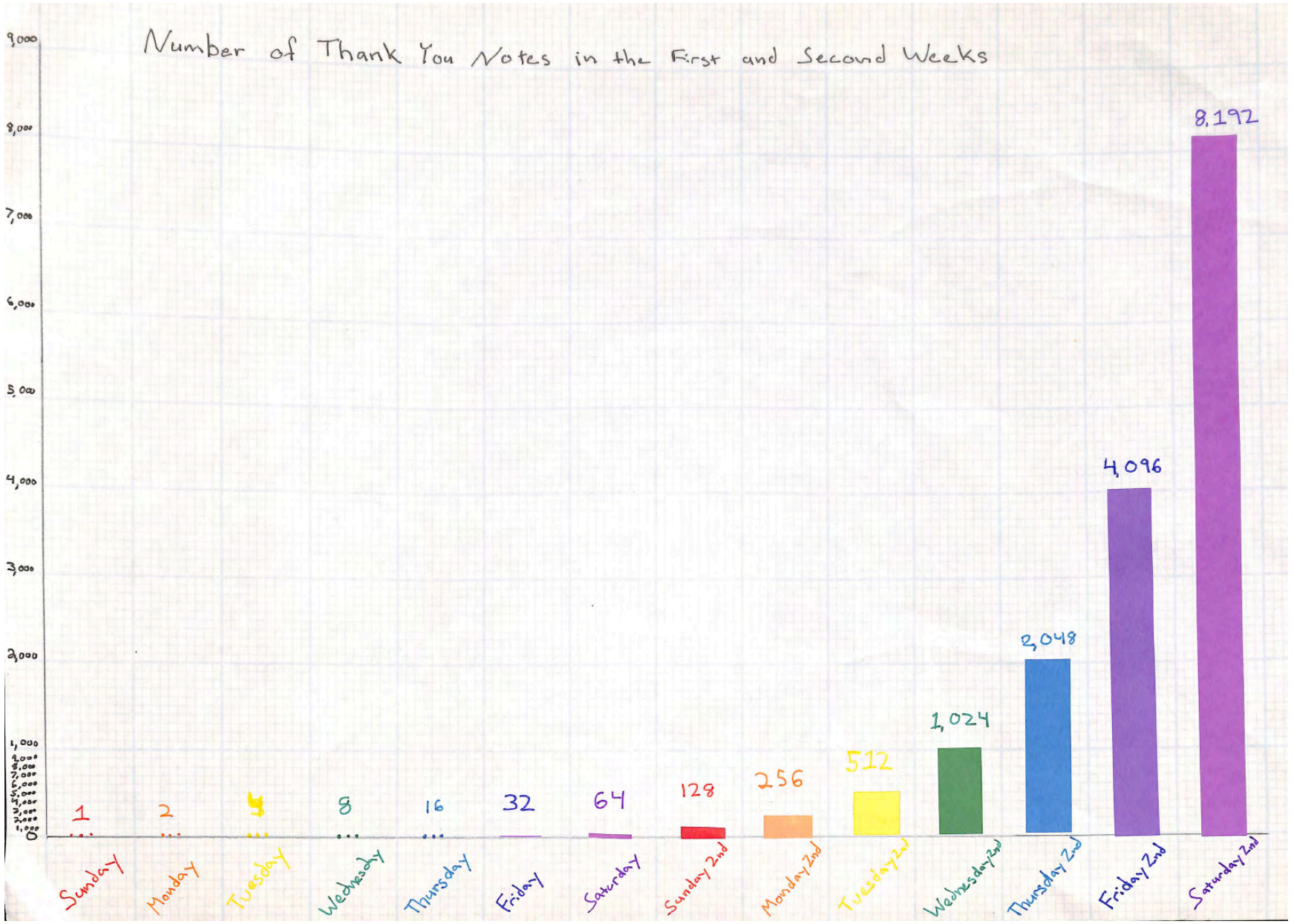
It worked!

This scale fits 9,000 thank notes on one piece of graph paper.

This is how he drew it.



"Wow, that's A LOT of thank you notes!!! I'm going to draw both weeks on one graph to see what it looks like."



What are some things you notice about the increase of thank you notes during the two weeks?

"Mom!" calls Jasmine, "Did you order the envelopes for next week yet?"

"I'm doing it right now. Makaila's dad said he ordered 127 envelopes for last week. Is that how many you want for next week?"

"We'll need a lot more than that, I'll just add it up now."

Days	N
Sunday	128
Monday	256
Tuesday	512
Wednesday	1024
Thursday	2048
Friday	4096
Saturday	8192

How many envelopes will they need for the second week?

Days	N
Sunday	128
Monday	256
Tuesday	512
Wednesday	1024
Thursday	2048
Friday	4096
Saturday	8192

128+256+512+1,024+2,048+4,096+8,192= 16,256!!!!!

"Mom, can you please order 16,256 envelopes?" asks Jasmine.

"What? That is way too many envelopes!! The link Makaila's dad sent me lists boxes of 100 envelopes. I can get you 4 boxes, but that's it."

Which day of the week will they run out of envelopes?

How many people do you know who will write thank you notes if you ask for their help?

Appendix: Additional Questions to Ponder…

Which day of the second week had almost the same amount of thank you notes as the whole first week?

Sunday

*In the whole first **week** 127 thank you notes are written; on the first **day** of the second week, 128 thank you notes are written.*

How many more envelopes did Jasmine's mom order than Makaila's dad?

273 more envelopes

4 x boxes of 100 envelopes= 400 envelopes. Makaila's dad ordered 127 envelopes. 400 – 127 = 273

Makaila's dad's envelopes were enough for 7 days in the first week. How many days did Jasmine's mom's envelopes last in the second week?

2 full days

128 + 256 = 384 thank you notes written on Sunday and Monday of the second week with only 400 – 384 = 16 available for Tuesday

How many people will Jasmine have asked if she asks a new person to help her write thank you notes each day during the two-week period?

13 people

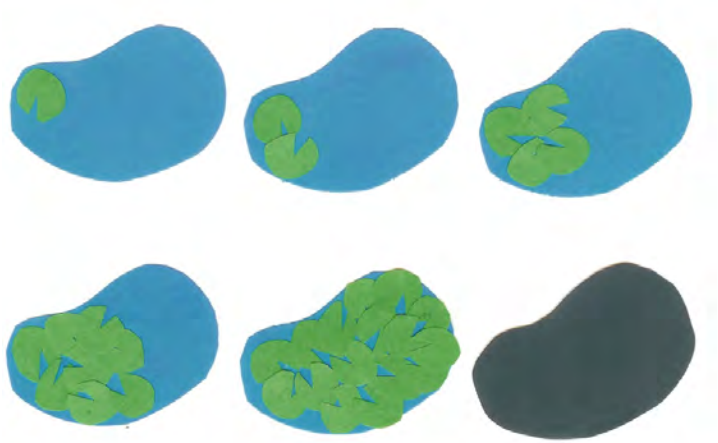
How many people did Jasmine's brother Marcus ask to help write thank you notes during the two-week period? Marcus started writing notes on Tuesday.

11 people

How many thank you notes could you and the people you asked to help write, if you all followed the same pattern of each person writing one thank you note and asking a new person to write one thank you note each day?

This story was inspired by a book, *The Limits of Growth:*

The authors included a French riddle to illustrate their topic, the initially slow and then rapid nature of exponential growth that can overwhelm limited resources. Here is a short version of the riddle: On Day 1, there is one lily pad in a pond. The number of lily pads doubles every day. If unchecked, the lily pads will cover the pond in 30 days, killing other life in the pond. If you decide to wait to clear the lily pads until they cover half the pond, what day will that be?



A common answer is half-way to the end of the month, but on Day 15 the area covered is still barely noticeable. If you wait until the lily pads cover half of the pond, you will be frantically clearing the pond on Day 29!

Meadows, D. H., D. L. Meadows, J. Randers, and W. W. Behrens, III. 1972. *The Limits to Growth.* New York: Universe Books, p. 29.

For a visual model of the lily pond riddle, see:
https://www.youtube.com/watch?v=0BSaMH4hINY

About the Authors:
Katrina van Zee and Emily van Zee

We wrote this book together while in quarantine on opposite coasts of the United States during the beginning of the COVID-19 pandemic. We had discussions on Zoom after remote sessions for a lab-based physics course for prospective elementary teachers that Emily was teaching. See: https://open.oregonstate.education/physicsforteachers/

Emily has long been an advocate for climate change education and has a fondness dating back years for the lily pad example of exponential growth and its limits. Katrina remembered this and started writing the story in a context reflecting her own teaching setting. Seeing a pattern of exponential growth in the infection rate for Covid-19 was disquieting, especially for Katrina who lives in the New York City area.

Prior to the coronavirus shutdown, Katrina, an informal science educator at Liberty Science Center, was teaching elementary students in a partnership school in an area that was one of the first clusters of Covid-19 infections in New Jersey. In thinking of these students and how to explore the concept of exponential growth in the context of this global pandemic, Katrina thought of using thank you notes to family, friends, neighbors, and essential workers as the vehicle for fostering understanding of the phenomenon of exponential growth.

The opening questions in the book: “What are some of the ways that people in your home help each other? What are some of the ways that you can think of to help your friends and neighbors? What are some of the jobs that people in our community do to help others?” are how Katrina would start a class discussion on this topic. The discussion begins with a story that is personal to each student, broadens to include friends and neighbors, and then expands to the essential workers and the whole community.

For more information, please contact us at
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